Spreading of Wavepackets in One-Dimensional Disordered Chains. Spreading Mechanisms

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Abstract

There are three different types of the evolution of a wavepacket in disordered nonlinear Schrödinger and anharmonic oscillator chains: **I)** localization as a transient, with subsequent subdiffusion; **II)** the absence of the transient and immediate subdiffusion; **III)** selftrapping of a part of the packet, and subdiffusion of the remainder. Here we focus on the mechanisms that explain subdiffusive spreading of the wavepacket which is due to weak but nonzero chaotic dynamics inside the packet. Chaos is a combined result of resonances and nonintegrability. As a result the mode outside the packet is heated by the packet nonresonatly. We estimate the number of resonant modes in the packet and study the nature of resonant mode pairs by performing a statistical numerical analysis. The predicted second moment of the packet is increasing as $t^{1/3}$ which is in a good agreement with our numerical findings.

Models

1

Model I: Disordered Nonlinear Schrödinger chain

Hamiltonian: $\mathcal{H}_D = \sum_l \epsilon_l |\psi_l|^2 + \frac{\beta}{2}|\psi_l|^4 - (\psi_{l+1}\psi_l^* + \psi_{l+1}^*\psi_l)$

ψ_l : complex variables

ϵ_l: on-site energies distributed uniformly in $\left[-\frac{W}{2}, \frac{W}{2}\right]$

DNLS equation:
$$\dot{\psi}_l = \partial\mathcal{H}_D/\partial(i\psi_l^*)$$
$$i\dot{\psi}_l = \epsilon_l\psi_l + \beta|\psi_l|^2\psi_l - \psi_{l+1} - \psi_{l-1}$$

Conserved quantities: energy and norm

Model II: Klein-Gordon chain [see Poster 24]

Hamiltonian: $\mathcal{H}_K = \sum_l \frac{p_l^2}{2} + \frac{\tilde{\epsilon}_l}{2}u_l^2 + \frac{1}{4}u_l^4 + \frac{1}{2W}(u_{l+1} - u_l)^2$

u_l, p_l: generalized coordinates and momenta

$\tilde{\epsilon}_l$: are chosen uniformly in $\left[\frac{1}{2}, \frac{3}{2}\right]$

KG equation:
$$\ddot{u}_l = -\partial\mathcal{H}_K/\partial u_l$$
$$\ddot{u}_l = -\tilde{\epsilon}_l u_l - u_l^3 + \frac{1}{W}(u_{l+1} + u_{l-1} - 2u_l)$$

Regimes of spreading

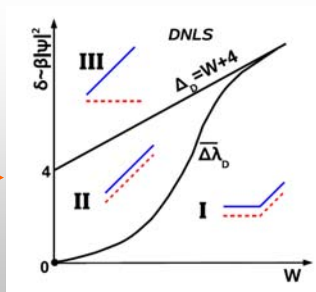
3

Single site excitation: $\psi_l = \delta_{l,l_0}, \epsilon_{l_0} = 0$

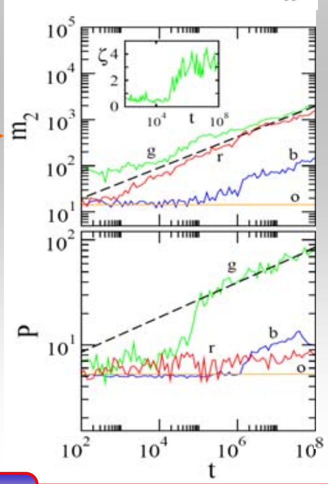
Frequency shift of a single site oscillator: $\delta_l = \beta|\psi_l|^2$

Three dynamical regimes: I) weak nonlinearity regime; II) intermediate regime [3] and III) selftrapping regime [4] with

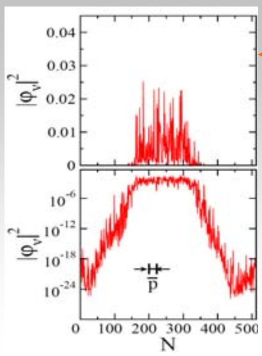
$\mathrm{I)}\ \delta_l < \overline{\Delta\lambda};\ \mathrm{II)}\ \overline{\Delta\lambda} < \delta_l < \Delta;\ \mathrm{III)}\ \Delta < \delta_l$ [5]



Schematic phase diagram. For each phase the dependence of $\log(m_2)$ (blue solid curves) and of log P (red dashed curves) versus log t are shown.



m_2 and P versus time in log-log plots with W=4 and β=0, 0.1, 1, 4.5 ((o)range,(b)lue, (g)reen, (r)ed). The disorder realization is kept unchanged. Dashed straight lines guide the eye for exponents 1/3 (m_2) and 1/6 (P). *Inset*: ς versus t for β=1.



Norm density distributions in the NM space at time $t=10^8$ for the initial excitations in the regime II (W=4, β=1). $|\varphi_\nu|^2$ is plotted in linear (logarithmic) scale in the upper (lower) plots. The maximal mean value of the localization volume of the NMs <p>~22 (shown schematically in the lower plots) is much smaller than the length over which the wave packet spreads.

Wave packet evolution (DNLS)

2

Linear problem $(\beta = 0)$ Ansatz: $\psi_l = A_l \exp(-i\lambda t)$

Eigenvalue problem : $\lambda A_l = \epsilon_l A_l - A_{l-1} - A_{l+1}$

Eigenvectors or normal modes (NMs): $A_{\nu,l}$

Eigenfrequency spectrum: $\lambda_\nu \in \left[-2 - \frac{W}{2}, 2 + \frac{W}{2}\right]$

Eigenvectors are localized (Anderson localization [1])

Width of the spectrum: $\Delta = W + 4$

Localization volume of NMs : $p_\nu = 1/\sum_l A_{\nu,l}^4$

Average spacing of eigenvalues of NMs within the range of localization volume : $\overline{\Delta\lambda} \approx \Delta \cdot W^2/360$

NMs are ordered according to $X_\nu = \sum_l l A_{\nu,l}^2$

Nonlinear problem

SABA symplectic integrators are used for numerical simulations [2]

General solution in NM space $\psi_l = \sum_\nu \phi_\nu A_{\nu,l}$

Normalized distribution: $z_\nu \equiv |\phi_\nu|^2/\sum_\mu |\phi_\mu|^2$

Participation number: $P = 1/\sum_\nu z_\nu^2$

Second moment: $m_2 = \sum_\nu (\nu - \bar{\nu})^2 z_\nu$ with $\bar{\nu} = \sum_\nu \nu z_\nu$

Compactness index: $\zeta = \frac{P^2}{m_2}$

Spreading mechanisms

4

Cold exterior is incoherently heated by the packet; part of the modes inside the packet evolve chaotic [5]

Mode-mode resonances inside the packet

Number of resonant modes inside the packet:

A NM with $|\phi_\nu|^2 = n_\nu$ will excite other modes in first order in βn_ν as

$|\phi_\mu| = \beta n_\nu R_{\nu,\mu}^{-1}|\phi_\nu|\ ,\ R_{\nu,\mu} \sim \left|\frac{\lambda_\nu - \lambda_\mu}{I_{\mu,\nu,\nu,\nu}}\right|$

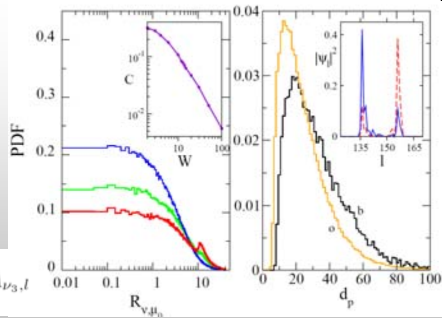
with the overlap integrals $I_{\nu,\nu_1,\nu_2,\nu_3} = \sum_l A_{\nu,l}A_{\nu_1,l}A_{\nu_2,l}A_{\nu_3,l}$

Resonance condition $R_{\nu,\mu} < \beta n_\nu$

Statistics: for a given ν mode obtain $R_{\nu,\mu_0} = \min_{\mu\neq\nu} R_{\nu,\mu}$

Important: $\mathcal{W}(R_{\nu,\mu_0} \to 0) \to C(W) \neq 0$

The probability for a mode *n* to be resonant with at least one other mode $\mathcal{P} = \int_0^{\beta n} \mathcal{W}(x)\mathrm{d}x$



Statistical properties of NMs.
Left plot: Probability densities $\mathcal{W}(R_{\nu,\mu_0})$ of NMs being resonant. Disorder strength W=4,7,10 (from top to bottom). *Inset:* The limiting value C versus W.

Right plot: Probability densities ***F***(d_p) of peak distances between resonant NM pairs for W=4 for different cutoffs $R_{\nu,\mu_0} < x_c \sim \beta n$; (o)range: x_c=0.1, (b)lack: x_c=0.01. *Inset*: The eigenvectors of a resonant pair of double-peaked states (solid and dashed lines).

Predicted growth of the second moment $m_2 \sim C^{2/3}\beta^{4/3}t^\alpha\ ,\ \alpha = 1/3$ is closed to the numerically observed one

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